Données haute fréquence s

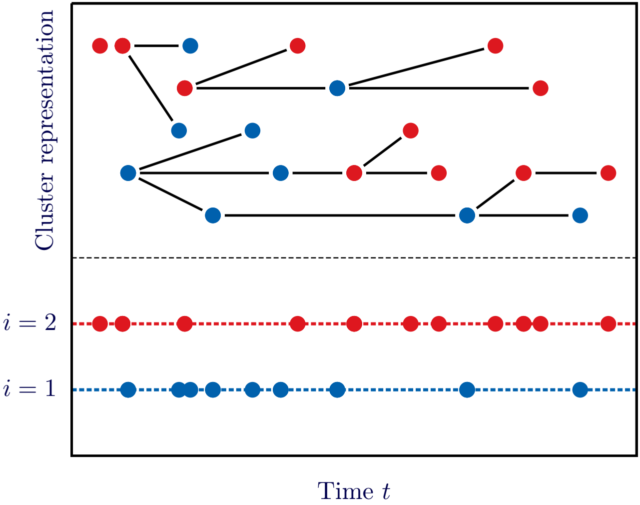
Analyse et modélisation statistique multi-échelle de séries chronologiques financières

Cours de Master - Probabilités et Finances - Sorbonne Université'

Slides de la partie IV

Applications avancées des processus Hawkes

Emmanuel Bacry
DR CNRS, CEREMADE, Université' Paris-Dauphine, PSL,
CSO, Health Data Hub
emmanuel.bacry@cnrs.fr



Cluster representation
$i = 2$
$i = 1$
Time t

For each component:

$$\Lambda^i = \mathbb{E}\left[\lambda(t)\right] = \mu_i + \sum_{j=1}^{D} \Lambda^j \|\phi^{ij}\|$$

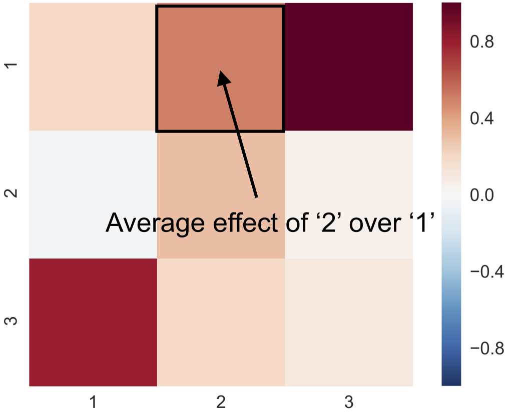
where we define

$$\|\phi^{ij}\| = \int_0^\infty \phi^{ij}(t)dt$$

Hence:

- μ^i is the immigrant intensity of type i events.
- $\frac{\Lambda^j}{\Lambda^i}\|\phi^{ij}\|$ is the fraction of type i events "triggered" by type j events.
- $\|\phi^{ij}\|$ is the average number of type i event triggered by a type j event.
- The shape of $\phi^{ij}(t)$ specifies how the excitation develops in time.

$$\boldsymbol{G} = \{G^{ij}\}_{i,j=1,\dots,D} = \{\|\phi_{ij}(t)\|\}_{i,j=1,\dots,D}$$



Provides a *summary* of the interactions

Inference in D-dimensionnal Hawkes processes framework

Estimating $D \times D$ real-valued functions

- **Parametric approaches**
  - ϕ^{ij} = linear combination of atomic functions in a dictionnary (e.g., exponential functions with various decay exponents)
  - Many procedures with various assumptions (sparsity, low-rank, ...)

- **Non-parametric approach**
  - Several methods in small dimension but **very difficult task in large dimension !**
  - M.Achab, et al. ICML (2017) JMLR (2017)
    $\rightarrow$ **Direct estimation of $(\boldsymbol{G})^{ij} = \int \phi^{ij}$ without estimation of ϕ^{ij}**

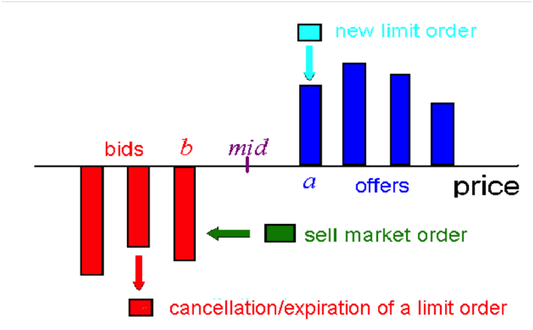
M.Achab, E.B., I.Mastromatteo, J.-F.Muzy, ICML (2017) JMLR (2017)

Idea

- **Direct estimation of $(\boldsymbol{G})^{ij} = \int \phi^{ij}$ without estimation of ϕ^{ij}**
- Actually, $\boldsymbol{G}$ encodes **Granger causality** between nodes!

$\Longrightarrow$ Cumulant matching method for estimation of $\boldsymbol{G}$

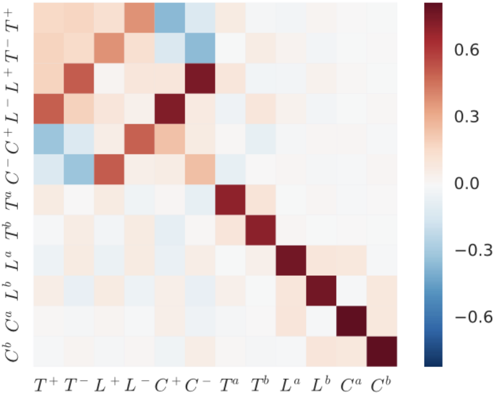
- Highly non-convex problem: polynomial or order 10 with respect to the entries of $(\boldsymbol{I} - \boldsymbol{G})^{-1}$
- Not so hard, local minima turns out to be good (deep learning literature, AdaGrad)
- We prove statistical consistency of the method



- Limit order : offer to buy (sell) a specific quantity at a certain price
- Cancellation : withdrawal of a limit order
- Market order : order to buy (sell) at best available price

M.Achab, E.B., J.-F.Muzy, M.Rambaldi, Quantit. Finance (2018)

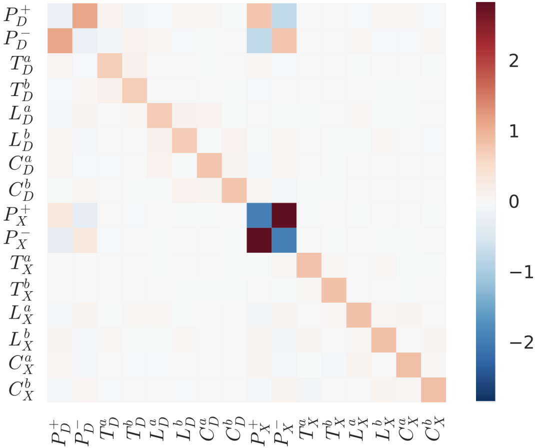
High-frequency financial data (DAX order book dynamics)
Estimation of 144 kernels



$\widehat{G}$

M.Achab, E.B., J.-F.Muzy, M.Rambaldi, Quantit. Finance (2018)

High-frequency financial data (DAX+Eurostoxx order book dynamics)
Estimation of 256 kernels



$\widehat{G}$

M.Achab, E.B., J.-F.Muzy, M.Rambaldi, Quantit. Finance (2018)

High-frequency financial data (DAX+Eurostoxx order book dynamics)
Estimation of 256 kernels

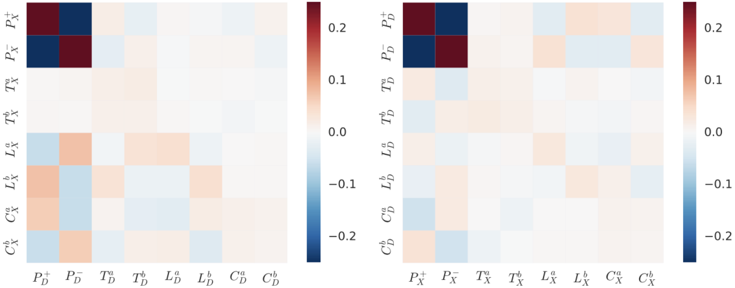


Figure 10. Submatrices of the Kernel norm matrix $\boldsymbol{G}$ corresponding to the effect of DAX events on EUROSTOXX STOXX events (left) and vice versa (right). These two submatrices correspond to the ones lying on the antidiagonal on the Figure 9

The Reaction matrix

$$\boldsymbol{R} = (\boldsymbol{I_d} - \boldsymbol{G})^{-1}$$

- gives the average direct and indirect effect of an event;
- R^{ij} = number of events of type i generated in total by an event of type j
- $\Lambda^i = E(\lambda_t^i) = \sum_j R^{ij}\mu_j$

Let δ_i be the mid-price change determined by an event of type i, then

$$\Delta_\tau P(t) \equiv P(t+\tau) - P(t) = \sum_{i \in M} \delta_i \int_t^{t+\tau} dN_s^i$$

And the volatility at time scale τ :

$$\sigma_\tau^2 = E(\Delta_\tau P^2) = \sum_{i,j \in M} \delta_i \delta_j \int_0^\tau \int_0^\tau E(dN_s^i dN_{s'}^j)$$

Putting together

- $\boldsymbol{R} = (\boldsymbol{I_d} - \boldsymbol{G})^{-1}$
- $\sigma_\tau^2 = \sum_{i,j \in M} \delta_i \delta_j \int_0^\tau \int_0^\tau E(dN_s^i dN_{s'}^j)$

One gets, after some calculations one obtains for the diffusive volatility :

$$\frac{\sigma_\tau^2}{\tau} \xrightarrow[\tau \to \infty]{} \sum_m \Lambda^m \xi_m^2 = \sum_{m=1}^{n} \Lambda^m \left(\sum_{i \in M} \delta_i R^{im} \right)^2$$

where

$$\xi_m = \text{average volatility per event of type } m$$

We have a link from microscopic dynamics to the diffusive regime

- $N_{i,\alpha}(t)$ counting process associated with actions α of agent i.
- We will suppose that $i = 1, \ldots, M$ and $\alpha \in \mathcal{A} = \{P^+, P^-, L^a, L^b, C^a, C^b, T^a, T^b\}$ where
  - P^+ (P^-) orders that immediately move upward (downward) the mid-price;
  - T^a (T^b) aggressive orders at the best ask (bid) that do not move the price;
  - L^a (L^b) new limit orders that arrive at the best ask (bid);
  - C^a (C^b) cancel orders at the best ask (bid) that do not move the price;
- $\phi^{i,\alpha;j,\beta}$ = influence of order type β of agent j on order type α of agent i

Total number of interactions: $(M \times 8) \times (M \times 8)$
For $M = 15$ agents that's 14400 kernels to estimate !!

Such huge number of kernels is hard to handle.

$\downarrow$

Work hypothesis:

- Influence on agent i from agent j actions does not depend on j provided $j \neq i$.
- That is

$$\phi^{i,\alpha;j,\beta}(t) = \begin{cases} \phi^{i,\alpha;\beta}\ \ (t) \text{ if } i \neq j \\ \phi^{i,\alpha;i,\beta}(t) \text{ if } i = j \end{cases}$$

Data are labelled data provided by Euronext

- CAC40 index future
- we consider the most liquid expiry for each day
- from March 1st 2016 to February 28th 2017;
- 111 unique members (connections are aggregated);
- focus on equity hours (08:00 - 16:30 London time)

We consider this subset of agents:

- at least 1000 orders at the first level;
- are active “uniformly” between 08:00 and 16:30;
- respect the above for at least 30 days.

Total number of agent considered $M = 16$

Name	Description
End of day (EOD) position	Absolute change in inventory at the end of the trading day, divided by the total volume traded by the agent.
Proprietary	Fraction of the orders that are market as proprietary by the agent.
Order lifetime	Median time between limit order insertion and cancellation/modification.
Inter-event time	Median time between two different orders by the same agent.
Limit-filled	Fraction of the submitted limit orders that are at least partially filled.
Canceled orders	Fraction of limit orders that are eventually canceled.
Aggressive volume	Ratio of the volume traded aggressively over the total traded volume by the agent.
Orders/Trades	Number of orders submitted for each trade.
Order size	Average order size (in contracts).
Time present at L1	Fraction of time the agent was present with a limit at at least one of the best quotes.
Present at both sides	Given the agent was present at the best, fraction of time he was present at both sides simultaneously.
Active connections	Average number of connections used by the agent per day.
Daily volume fraction	Fraction of the total traded volume (total buy + total sell) in which the agent is involved.

	240	140	478	127	636	398	503	274	566	59	584	364	597	455	244	669
EOD Position / Volume (%)	0.00	0.01	0.15	3.73	3.83	4.54	9.71	14.9	16.2	22.3	18.3	22.7	24.5	29.1	28.2	32.8
% Proprietary	100.0	100.0	100.0	100.0	100.0	100.0	0.22	68.1	97.8	100.0	1.19	100.0	2.10	98.7	0.00	0.37
Order lifetime (s)	0.51	0.61	0.20	3.57	0.99	0.33	1.33	3.19	42.0	4.14	7.87	5.17	4.32	3.04	6.31	11.1
Inter-event time (s)	0.01	0.00	0.02	0.01	0.00	0.06	0.63	0.07	0.63	0.01	1.64	0.01	1.65	0.12	2.45	2.33
Limit filled (%)	5.09	6.15	8.40	10.5	6.35	10.5	28.3	19.8	47.9	1.58	50.4	5.45	42.4	4.05	23.5	42.0
Limit (%)	51.1	50.0	48.9	44.3	36.3	37.7	49.3	47.5	53.6	40.7	31.0	51.0	54.1	50.0	48.1	53.4
Cancel (%)	48.4	47.2	46.2	40.0	33.7	33.9	36.2	37.9	27.9	40.1	14.5	48.3	31.1	48.0	36.6	30.2
Replace (%)	0.00	0.08	3.43	13.6	29.4	27.4	6.58	8.78	7.54	18.4	40.1	0.04	5.77	1.57	11.1	8.57
Aggressive (%)	0.51	2.69	1.42	2.08	0.60	1.01	7.97	5.76	10.9	0.80	14.4	0.62	9.08	0.43	4.25	7.78
Aggressive volume (%)	14.9	64.0	34.0	34.4	15.0	13.2	49.9	46.9	37.4	56.2	44.4	25.0	34.8	27.0	27.0	28.8
Orders/Trades (%)	3994.2	1085.8	1351.5	1128.0	5573.1	1036.1	238.5	524.7	190.3	5276.6	191.9	2609.4	206.5	3915.7	162.6	497.4
Order size (contracts)	1.02	1.38	2.33	1.65	1.15	4.41	2.45	1.64	1.70	1.88	3.66	2.42	2.70	4.08	3.75	2.38
Time present at L1 (%)	76.8	99.4	51.1	87.6	73.7	26.5	39.3	38.4	22.7	30.4	19.7	36.1	25.1	22.2	27.0	42.6
Present at both sides (%)	39.1	69.1	9.21	36.9	25.9	0.69	4.71	5.07	1.59	1.61	0.99	1.32	1.75	0.69	1.91	5.87
Active connections	19.9	98.2	16.2	32.2	19.6	2.16	18.9	19.8	9.32	5.47	17.7	10.5	4.26	13.9	2.55	3.69
Daily volume fraction (%)	2.22	31.3	4.68	6.30	1.28	3.59	6.05	5.63	2.04	4.76	3.85	2.00	1.88	2.13	2.65	2.73

	240	140	478	127	636	398	503	274	566	59	584	364	597	455	244	669
EOD Position / Volume (%)	0.00	0.01	0.15	3.73	3.83	4.54	9.71	14.9	16.2	22.3	18.3	22.7	24.5	29.1	28.2	32.8
% Proprietary	100.0	100.0	100.0	100.0	100.0	100.0	0.22	68.1	97.8	100.0	1.19	100.0	2.10	98.7	0.00	0.37
Order lifetime (s)	0.51	0.61	0.20	3.57	0.99	0.33	1.33	3.19	42.0	4.14	7.87	5.17	4.32	3.04	6.31	11.1
Inter-event time (s)	0.01	0.00	0.02	0.01	0.00	0.06	0.63	0.07	0.63	0.01	1.64	0.01	1.65	0.12	2.45	2.33
Limit filled (%)	5.09	6.15	8.40	10.5	6.35	10.5	28.3	19.8	47.9	1.58	50.4	5.45	42.4	4.05	23.5	42.0
Limit (%)	51.1	50.0	48.9	44.3	36.3	37.7	49.3	47.5	53.6	40.7	31.0	51.0	54.1	50.0	48.1	53.4
Cancel (%)	48.4	47.2	46.2	40.0	33.7	33.9	36.2	37.9	27.9	40.1	14.5	48.3	31.1	48.0	36.6	30.2
Replace (%)	0.00	0.08	3.43	13.6	29.4	27.4	6.58	8.78	7.54	18.4	40.1	0.04	5.77	1.57	11.1	8.57
Aggressive (%)	0.51	2.69	1.42	2.08	0.60	1.01	7.97	5.76	10.9	0.80	14.4	0.62	9.08	0.43	4.25	7.78
Aggressive volume (%)	14.9	64.0	34.0	34.4	15.0	13.2	49.9	46.9	37.4	56.2	44.4	25.0	34.8	27.0	27.0	28.8
Orders/Trades (%)	3994.2	1085.8	1351.5	1128.0	5573.1	1036.1	238.5	524.7	190.3	5276.6	191.9	2609.4	206.5	3915.7	162.6	497.4
Order size (contracts)	1.02	1.38	2.33	1.65	1.15	4.41	2.45	1.64	1.70	1.88	3.66	2.42	2.70	4.08	3.75	2.38
Time present at L1 (%)	76.8	99.4	51.1	87.6	73.7	26.5	39.3	38.4	22.7	30.4	19.7	36.1	25.1	22.2	27.0	42.6
Present at both sides (%)	39.1	69.1	9.21	36.9	25.9	0.69	4.71	5.07	1.59	1.61	0.99	1.32	1.75	0.69	1.91	5.87
Active connections	19.9	98.2	16.2	32.2	19.6	2.16	18.9	19.8	9.32	5.47	17.7	10.5	4.26	13.9	2.55	3.69
Daily volume fraction (%)	2.22	31.3	4.68	6.30	1.28	3.59	6.05	5.63	2.04	4.76	3.85	2.00	1.88	2.13	2.65	2.73

At the far left : Flat position, fast, high order to trade ratio, proprietary, high presence.
$\simeq$ **Market maker**

	240	140	478	127	636	398	503	274	566	59	584	364	597	455	244	669
EOD Position / Volume (%)	0.00	0.01	0.15	3.73	3.83	4.54	9.71	14.9	16.2	22.3	18.3	22.7	24.5	29.1	28.2	32.8
% Proprietary	100.0	100.0	100.0	100.0	100.0	100.0	0.22	68.1	97.8	100.0	1.19	100.0	2.10	98.7	0.00	0.37
Order lifetime (s)	0.51	0.61	0.20	3.57	0.99	0.33	1.33	3.19	42.0	4.14	7.87	5.17	4.32	3.04	6.31	11.1
Inter-event time (s)	0.01	0.00	0.02	0.01	0.00	0.06	0.63	0.07	0.63	0.01	1.64	0.01	1.65	0.12	2.45	2.33
Limit filled (%)	5.09	6.15	8.40	10.5	6.35	10.5	28.3	19.8	47.9	1.58	50.4	5.45	42.4	4.05	23.5	42.0
Limit (%)	51.1	50.0	48.9	44.3	36.3	37.7	49.3	47.5	53.6	40.7	31.0	51.0	54.1	50.0	48.1	53.4
Cancel (%)	48.4	47.2	46.2	40.0	33.7	33.9	36.2	37.9	27.9	40.1	14.5	48.3	31.1	48.0	36.6	30.2
Replace (%)	0.00	0.08	3.43	13.6	29.4	27.4	6.58	8.78	7.54	18.4	40.1	0.04	5.77	1.57	11.1	8.57
Aggressive (%)	0.51	2.69	1.42	2.08	0.60	1.01	7.97	5.76	10.9	0.80	14.4	0.62	9.08	0.43	4.25	7.78
Aggressive volume (%)	14.9	64.0	34.0	34.4	15.0	13.2	49.9	46.9	37.4	56.2	44.4	25.0	34.8	27.0	27.0	28.8
Orders/Trades (%)	3994.2	1085.8	1351.5	1128.0	5573.1	1036.1	238.5	524.7	190.3	5276.6	191.9	2609.4	206.5	3915.7	162.6	497.4
Order size (contracts)	1.02	1.38	2.33	1.65	1.15	4.41	2.45	1.64	1.70	1.88	3.66	2.42	2.70	4.08	3.75	2.38
Time present at L1 (%)	76.8	99.4	51.1	87.6	73.7	26.5	39.3	38.4	22.7	30.4	19.7	36.1	25.1	22.2	27.0	42.6
Present at both sides (%)	39.1	69.1	9.21	36.9	25.9	0.69	4.71	5.07	1.59	1.61	0.99	1.32	1.75	0.69	1.91	5.87
Active connections	19.9	98.2	16.2	32.2	19.6	2.16	18.9	19.8	9.32	5.47	17.7	10.5	4.26	13.9	2.55	3.69
Daily volume fraction (%)	2.22	31.3	4.68	6.30	1.28	3.59	6.05	5.63	2.04	4.76	3.85	2.00	1.88	2.13	2.65	2.73

At the far right :Slower, directional, lower order/trade ratio.
$\simeq$ **Directional agent**

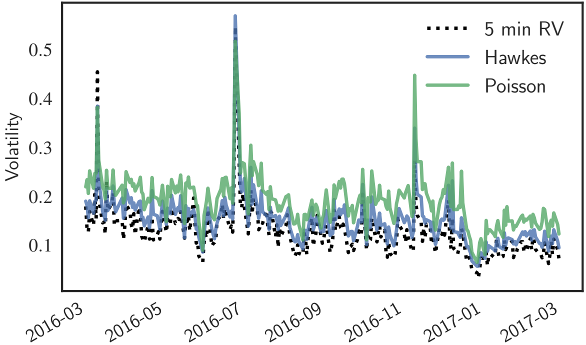
Average direct and indirect contribution to the total volatility of a single event of type $\{i, \alpha\}$:

$$\xi_{i,\alpha}^2 = \left(\sum_j \sum_\beta \delta_{j,\beta} R^{j,\beta;i,\alpha} \right)^2$$

where we assume that $\delta_{j,\beta} = 0$ if $\beta \notin \{P^+, P^-\}$.

And the total diffusive volatility writes

$$\sigma^2 = \sum_{i,\alpha} \Lambda^{i,\alpha} \xi_{i,\alpha}^2$$



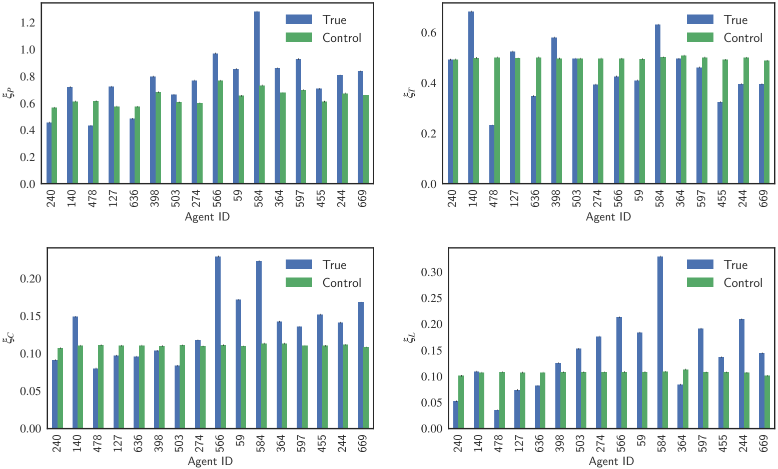
Hawkes volatility $= \sum_{i,\alpha} \Lambda^{i,\alpha} \xi_{i,\alpha}^2$
Poisson volatility $= \sum_{i,\alpha} \Lambda^{i,\alpha} \delta_{i,\alpha}^2$

We construct a control result to compare with.

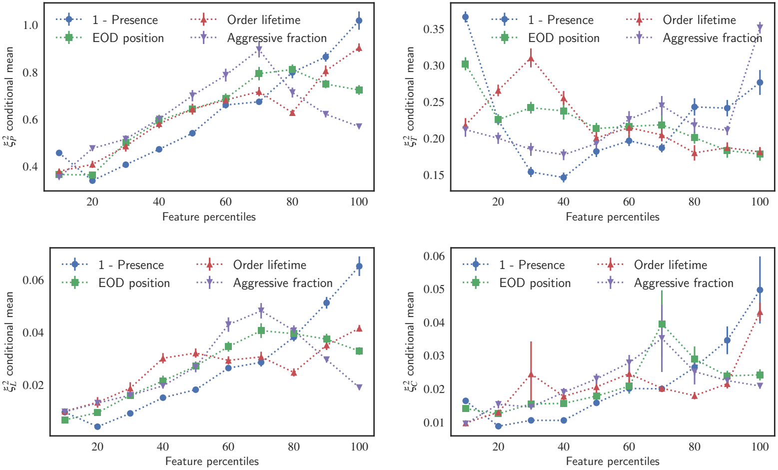
For each agent, each day the control has:

- Same number of orders.
- Same order composition.
- But agents labels are shuffled: orders are randomly assigned to agents.

$\Longrightarrow$ **differences from control are mainly due to timing.**



ξ strongly depends on the agent order timing



Market maker like agents have smaller impact per passive event

Given

$$\sigma^2 = \sum_{i,\alpha} \Lambda^{i,\alpha} \left(\sum_{j,\beta} \delta_{j,\beta} R^{j,\beta;i,\alpha} \right)^2$$

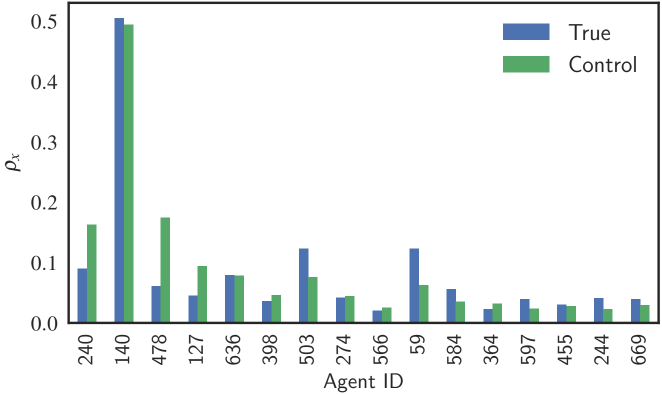
and

$$\Lambda^{i,\alpha} = \sum_{k,\gamma} R^{i,\alpha;k,\gamma} \mu^{k,\gamma},$$

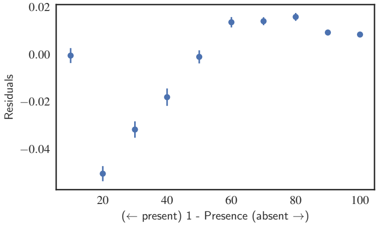
we define ρ_m for agent m as

$$\sigma^2 \rho_m = \sigma^2 - \sum_{i \neq m} \sum_{k \neq m} \sum_{\alpha,\gamma} R^{i,\alpha;k,\gamma} \mu^{k,\gamma} \left(\sum_{j \neq m} \sum_{\beta} \delta_{j,\beta} R^{i,\alpha;j,\beta} \right)^2$$

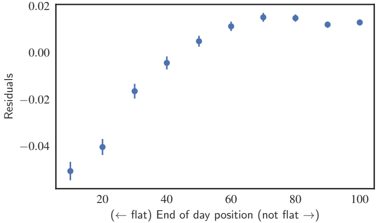
ρ_m **: Relative difference in volatility we would observe if we removed all the activity directly or indirectly generated by agent** x**.**

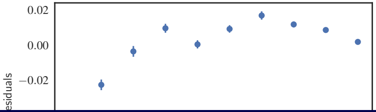


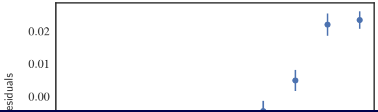
Significant differences with the control for most agents (and $\rho_m > 0$)



Plotting the residuals : $\rho_m - \rho_m^{\text{control}}$

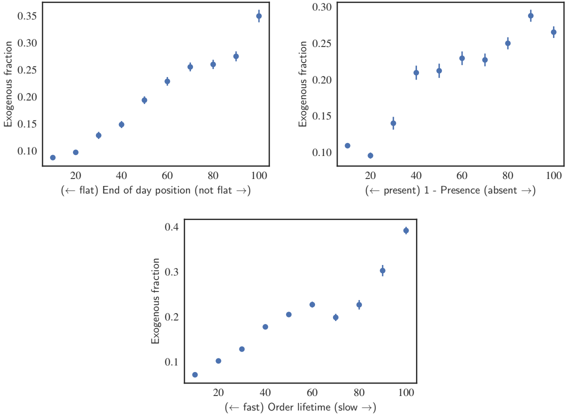






Exogenous fraction f_m for agent m

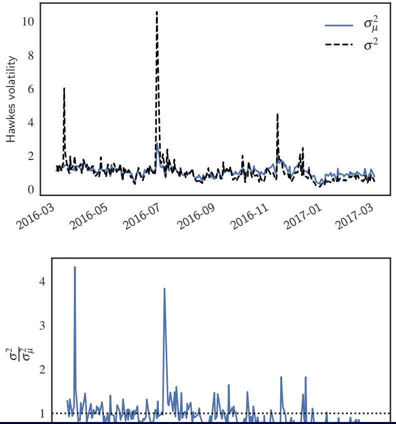
$$f_m = \frac{\sum_\alpha \mu^{m,\alpha}}{\sum_\alpha \Lambda^{m,\alpha}}.$$



Marker maker like (left side) are more "endogenous".

$$\sigma^2 = \sum_{i,\alpha} \Lambda^{i,\alpha} \left(\sum_{j,\beta} \delta_{j,\beta} R^{j,\beta;i,\alpha} \right)^2 = \sum_{i,\alpha} \mu^{i,\alpha} u^{i,\alpha} \approx \sum_{i,\alpha} \mu^{i,\alpha} \bar{u}^{i,\alpha} = \sigma_\mu^2$$

$u^{i,\alpha}$ = volatility per exogeneous event
$\bar{u}^{i,a}$ = mean value over the whole sample.



M.Bompaire, P.Deegan, S.Gaiffas, S.Poulsen, E.B., ...

- Python 3 et C++11
- Open-source (BSD-3 License)
- `pip install tick` (on MacOS and Linux...)
- `https://x-datainitiative.github.io/tick`
- Statistical learning for time-dependent models
- Point processes (Poisson, Hawkes), Survival analysis, GLMs (parallelized, sparse, etc.)
- A strong simulation and optimization toolbox
- Partnership with Intel (use-case for new processors with 256 cores)
- Many contributors
- New contributors are welcome !

Software: tick library

tick 0.1 Home Examples API Browse ▾ Search

tick

tick a machine learning library for Python 3. The focus is on statistical learning for time dependent systems, such as point processes. Tick features also tools for generalized linear models, and a generic optimization toolbox.

The core of the library is an optimization module providing model computational classes, solvers and proximal operators for regularization. It comes also with inference and simulation tools intended for end-users.

Show me »

Examples

Examples of how to simulate models, use the optimization toolbox, or use user-friendly inference tools.

Inference

User-friendly classes for inference of models

Simulation

User-friendly classes for simulation of data

Optimization

The core module of the library: an optimization toolbox consisting of models, solvers and prox (penalization) classes. Almost all of them can be combined